[Proceeding] Error exponent analysis for MIMO multiple-scattering channels

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***Abstract*—In this work, we derive closed-form expression for the Gallager's random coding error exponent for a MIMO multiple-scattering channel. The number of scattering stages is arbitrary but finite, white noise is present at the destination.**

I. INTRODUCTION

Random matrix products arise in multi-antenna channels modeling since earliest works on the topic, rigorously representing progressive scattering phenomena [1], and later on modeling concatenated transmit systems helped by multiple-antenna equipped relays [2], [3, and references therein] at high Signal to Noise Ratio (SNR). Relying on very recent results from random matrix theory and polynomial ensembles [4], in this work we move a step toward a full characterization of wireless systems whose channel matrix is suitably modeled by a product of several random matrices of finite size[1]. Focusing on a communication impaired by uncorrelated Rayleigh fading, we assume that only the destination is provided with statistical channel state information (CSI), leaving the more involved case of neither transmitter nor receiver aware of CSI for future investigation. We analyze for this channel the trade-off between system performance and required coding length at a prescribed rate below the channel capacity, i.e. we provide expression for the Gallager's lower bound to the error exponent of a MIMO system whose channel matrix is the product of an arbitrary number, say M, of independent rectangular matrices with standard Gaussian i.i.d. entries. The analysis straightforwardly generalizes to the case of independent matrices with zero-mean i.i.d. Gaussian entries, but with possibly different variances across the matrix factors. This models both a Rayleigh-faded, multiple-scattering channel with uncorrelated scatterers and possibly different scattering power, as well as a multi-hop MIMO relay channel with Uniform Power Allocation (UPA) at each relay stage, non-noisy relays and noisy received signal. Error exponent evaluation for MIMO systems in presence of receiver CSI has been carried out first[2] in the seminal paper [7], for Rayleigh fading channels with arbitrary (separable) spatial correlation at either link end. Rayleigh-product channels have been later investigated in [8] for the dual-hop case. To the best of the author knowledge, this is the first investigation assuming an arbitrary number of scattering stages.

[1]Indeed, the only closed-form result on mutual information of multiple-antenna systems with progressive scattering in the finite-dimensional case appears outside wireless information theoretic literature, in [5].

[2]In [6], where first the problem was set down, there is no final analytical expression for the error exponent.

We stress that, without CSI at either link end, the unique available result is for MIMO Rayleigh channels, and is derived in [9].

II. SYSTEM MODEL

Let us consider a channel represented by a random $n_r \times n_t$ matrix $\mathbf{H}$, having the following expression

$$\mathbf{H} = \prod_{i=0}^{M-1} \mathbf{H}_i \,.$$

where $\mathbf{H}_i$ is a $(n_r+\nu_{i-1})\times(n_r+\nu_i)$ random matrix with i.i.d. Gaussian entries and $\nu_0 = 0$. The matrix $\mathbf{H}$ models a M-stages multiple-scattering channel, affected by Rayleigh fading, with uniform power allocation (UPA) at the source and AWGN at the receiver

Assuming a coding length n_c for the transmitted signal and collecting the output of n_b successive channel uses, where n_b is the block-length of the fading process, the output signal can be expressed as

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{N}\,, \tag{1}$$

with $\mathbf{Y}$ $n_r \times n_c$ matrix-valued output, $\mathbf{X}$ $n_t \times n_c$ matrix-valued output and $\mathbf{N}$ AWGN matrix of size $n_r \times n_c$.

III. INFORMATION-THEORETIC ANALYSIS

Error exponent relates the achievable error probability of a coding strategy with the required coding length. While the rigorous definition of error exponent accounts for the exploitation of the optimal (in term of achievable error probability) code, i.e.

$$E(R) = \lim \sup_{N\to\infty} -\ln \frac{P_e^{\rm opt}(R,N)}{N},$$

with R the rate and N the coding length corresponding to the optimal error probability, due to the difficulty in evaluating it even for scalar channels, we resort to classical Gallager's lower bound for random coding, which leads to the evaluation of

$$E_R(p(\mathbf{X}), R, n_c) = \max_{0\le\rho\le 1}\left\{\max_{r\ge 0} E_0(p(\mathbf{X}),\rho,r,n_c) - \rho R\right\}, \tag{2}$$

with

$$E_0(p(\mathbf{X}),\rho,r,n_c) = -\frac{1}{n_c}\ln\mathcal{E}\,, \tag{3}$$

$\mathcal{E}$ denoting the following matrix integral

$$\int_{\mathbf{H}} p(\mathbf{H}) \int_{\mathbf{Y}} \left(\int_{\mathbf{X}} p(\mathbf{X}) p(\mathbf{Y}|\mathbf{X},\mathbf{H})^{\frac{1}{1+\rho}} e^{r\left[\mathrm{Tr}\mathbf{X}\mathbf{X}^{\dagger} - n_b \mathcal{P}\right]} d\mathbf{X} \right)^{1+\rho} d\mathbf{Y} d\mathbf{H} \quad (4)$$

Notice that (3) relies on the assumption that CSI is made available at the receiver, henceforth $p(\mathbf{Y}|\mathbf{X},\mathbf{H})$ is exploited in the calculus. Notice further that the optimal input w.r.t. the error exponent is the one which maximizes $E_R(p(\mathbf{X}), R, n_c)$, but for sake of simplicity we adopt hereinafter, as usual in the literature, the average power constrained capacity-achieving (in ergodic sense) distribution for $\mathbf{X}$, i.e. we resort to UPA.

Under the abovementioned assumptions, we can state the following

Theorem 3.1: The random coding bound on the error probability for ML deconding over a block-fading channel can be written as [7, Eq. (9)]

$$P_e \leq \alpha \exp\{-n_b n_c\} E_R(p(\mathbf{X}), R, n_c)\,, \quad (5)$$

with E_R defined in (2) and where E_0 from (3) can be written as

$$E_0(p(\mathbf{X}), \rho, r, n_c) = n_t \gamma (1+\rho) - n_r \ln(1+\rho) - \frac{1}{n_c} \ln|\mathbf{Z}|\,, \quad (6)$$

with $\gamma = \mathcal{P}/n_t$, $\mathcal{P}$ being the overall transmit power, and

$$\mathbf{Z}_{i,j} = \int_0^{+\infty} \lambda^{j-1} G_{0,M}^{M,0}\left(\begin{matrix} - \\ \nu_M, \ldots, \nu_2, \nu_1 + i - 1 \end{matrix} \middle| \lambda \right) \xi(\lambda) d\lambda$$

and $\xi(\lambda)$ an algebraic function of λ, a marginal unordered singular value of $\mathbf{H}$, whose joint distribution is characterized in [4].

Proof. See extended version.

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